

Stefano Mancini and Rodolfo Bonifacio

INFN, Dipartimento di Fisica, Università di Milano, Via Celoria 16, I-20133 Milano, Italy

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We study the effects of time uncertainty in the interaction of atoms with a standing light wave. We discuss its physical origin and the possibility to observe intrinsic decoherence effects by measuring the atomic momentum distribution.

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I. INTRODUCTION

Typically the decoherence effects are ascribed to the action of an environment, i.e. not controlled degrees of freedom. This is the so called environment-induced decoherence [1]. However, there could also be the possibility of an *intrinsic* nature of decoherence, i.e. coming from the system itself and not from external degrees of freedom, as pointed out recently [2]. The idea underlying this approach is the fact that the interaction time, i.e. the time interval in which the effective Hamiltonian evolution takes place, is a random variable [3]. This randomness can have different origins depending on the studied system.

In atom optics [4], for instance, the interaction time is determined by the transit time of the atoms through a cavity [5]. Nevertheless, this interaction time is random, due to eventual classical fluctuations of atomic velocities, but beyond that, due to the finite extend of the interacting atomic wave packet [6]. In fact, the instant at which a wave packet passes a given point of the axis is not determined precisely, but carries an uncertainty proportional to its width [6]. Hence, there should be purely quantum mechanical intrinsic decoherence effects. Here, we study the appearance of such effects in atomic diffraction by standing light wave.

On the other hand, decoherence effects relevant to quantum optics domain are now beginning to be tested experimentally [7]. As far as quantum objects bearing mass are concerned, decoherence has been only investigated for the motional states of the ions [8]. Then, it would be also interesting to experimentally verify such a new type of decoherence. To this end, we show a possible modification of the measurable atomic momentum distribution.

II. TIME FORMALISM

In this section we review the formalism describing non-dissipative decoherence derived in Refs. [3]. It is based on the idea that time is a random variable or, alternatively, that the system Hamiltonian (therefore its eigenvalues) fluctuates. This leads to random phases in the energy eigenstates representation. Then, the resulting evolution of the system must be averaged on a suitable probability distribution, and this leads to the decay of the off-diagonal elements of the density operator.

Let us consider an initial state $\rho(0)$ and consider the case of a random evolution time. Then, the evolved state will be averaged over a probability distribution $P(t, t')$, i.e.

$$\bar{\rho}(t) = \int_0^\infty dt' P(t, t') \rho(t'), \quad (1)$$

where $\rho(t') = \exp\{-iLt'\}\rho(0)$ is the usual solution of the Liouville-Von Neumann equation with $L \dots = [H, \dots]/\hbar$. One can write as well

$$\bar{\rho}(t) = V_L(t)\rho(0), \quad (2)$$

where

$$V_L(t) = \int_0^\infty dt' P(t, t') e^{-iLt'}. \quad (3)$$

In Ref. [3], the function $P(t, t')$ has been determined to satisfy the following conditions: i) $\bar{\rho}(t)$ must be a density operator, i.e. it must be self-adjoint, positive-definite, and with unit-trace. This leads to the condition that $P(t, t')$

must be non-negative and normalized, i.e. a probability density in t' , so that Eq.(1) is a completely positive map; ii) $V_L(t)$ satisfies the semigroup property $V_L(t_1 + t_2) = V_L(t_1)V_L(t_2)$, with $t_1, t_2 \geq 0$. These requirements are satisfied by

$$V_L(t) = \frac{1}{(1 + iL\tau)^{t/\tau}}, \quad (4)$$

and

$$P(t, t') = \frac{1}{\tau} \frac{e^{-t'/\tau}}{\Gamma(t/\tau)} \left(\frac{t'}{\tau}\right)^{(t/\tau)-1}, \quad (5)$$

where the parameter τ naturally appears as a scaling time. The expression (5) is the so-called Γ -distribution function, well known in line theory. Its interpretation is particularly simple when $t/\tau = r$ with r integer; in that case $P(r, t')$ gives the probability density that the waiting time for r independent events is t' and τ is the mean time interval between two events. Generally, the meaning of the parameter τ can be understood by considering the mean of the evolution time $\langle t' \rangle = t$, and its variance $\langle t'^2 \rangle - \langle t' \rangle^2 = \tau t$.

When $\tau \rightarrow 0$, $P(t, t') \rightarrow \delta(t - t')$ so that $\bar{\rho}(t) \equiv \rho(t)$ and $V_L(t) = \exp\{-iLt\}$ is the usual unitary evolution. However, for finite τ , the evolution operator $V_L(t)$ describes a decay of the off diagonal matrix elements in the energy representation, whereas the diagonal matrix elements remain constants, i.e. the energy is still a constant of motion. In fact, in the energy eigenbasis, Eqs.(2) and (4) yield

$$\bar{\rho}_{n,m}(t) = e^{-\gamma_{n,m}t} e^{-i\nu_{n,m}t} \rho_{n,m}(0), \quad (6)$$

where

$$\gamma_{n,m} = \frac{1}{2\tau} \log(1 + \omega_{n,m}^2 \tau), \quad (7)$$

$$\nu_{n,m} = \frac{1}{\tau} \arctan(\omega_{n,m} \tau), \quad (8)$$

with $\hbar\omega_{n,m} = (E_n - E_m)$ the energy difference. One can recognize in Eq.(6), beside the exponential decay, a frequency shift of every oscillating term.

The phase diffusion aspect of the present approach can also be seen in the evolution equation for the averaged density matrix $\bar{\rho}(t)$. Indeed, by differentiating with respect to time Eq.(2) and using (4) one gets the following master equation for $\bar{\rho}(t)$

$$\frac{d}{dt}\bar{\rho}(t) = -\frac{1}{\tau} \log(1 + iL\tau) \bar{\rho}(t). \quad (9)$$

It is worth noting that by expanding the logarithm up to second order in τ , one obtains

$$\frac{d}{dt}\bar{\rho}(t) = -\frac{i}{\hbar} [H, \bar{\rho}(t)] - \frac{\tau}{2\hbar^2} [H, [H, \bar{\rho}(t)]] , \quad (10)$$

which is the well known phase-destroying master equation [9]. Hence, Eq.(9) appears as a generalized phase destroying master equation taking into account higher order terms in τ . Nonetheless, the present approach is different from the usual master equation approach, in the sense that it is model independent, non perturbative and without specific statistical assumptions. In fact, the probability distribution (5), is derived only from the semigroup condition, and it is interesting to note that this condition yields a gaussian probability distribution as a limiting case when $t \gg \tau$.

III. ATOMIC DIFFRACTION BY STANDING LIGHT WAVE

We consider the deflection of an atomic beam of two-level atoms by a classical standing wave in a cavity as in Fig.(1). The atomic beam is normally incident on the standing wave and experiences an exchange of momentum with the photons in the light wave. We shall assume that the frequency of the light field is well detuned from the atomic resonance so that we may neglect the spontaneous emission. The Hamiltonian describing the system is [9]

$$H = \hbar\omega_0\sigma_Z + \frac{\mathbf{P}^2}{2M} + \hbar\Omega (\sigma_- e^{-i\omega t} + \sigma_+ e^{i\omega t}) \cos(kX), \quad (11)$$

where $\mathbf{P} \equiv (P_X, P_Y, P_Z)$ is the center of mass momentum of the atom, M is the atomic mass, σ_Z and σ_{\pm} are the pseudo spin operators for the atom, ω_0 and ω are the atomic and field frequencies, $k = \omega/c$ is the wave number of the standing wave, and Ω the Rabi frequency.

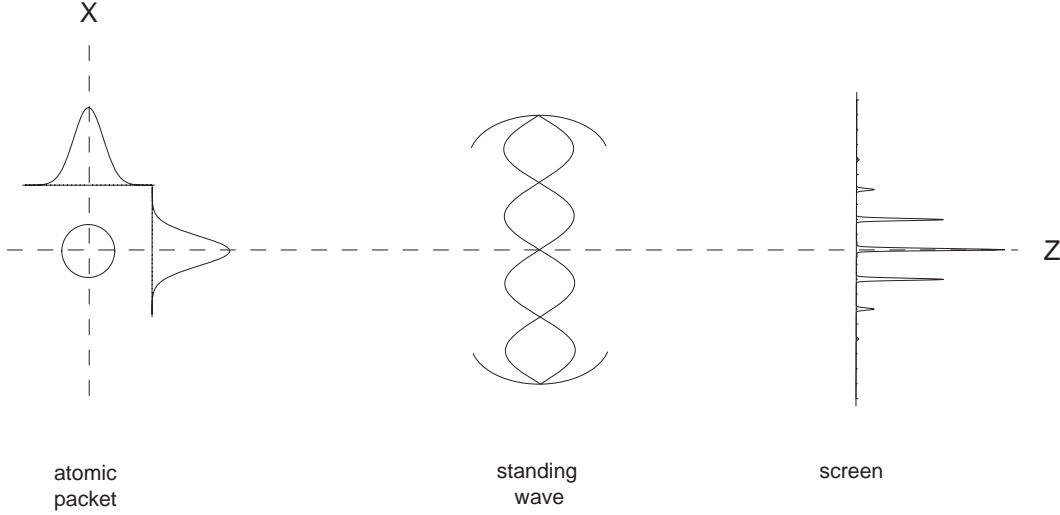


FIG. 1. Scheme

We shall assume that the interaction time is sufficiently small that the transverse kinetic energy absorbed by the atom during the interaction can be neglected (Raman-Nath approximation). This is equivalent to neglect the term $P^2/2M$ in the Hamiltonian (11). Then, transforming to the interaction picture (with $H_0 = \hbar\omega\sigma_Z$) the Hamiltonian may be written in the form

$$H = \hbar\Delta\sigma_Z + 2\hbar\Omega\sigma_X \cos(kX), \quad (12)$$

$$= \hbar V(X) [\cos\theta(X)\sigma_Z + \sin\theta(X)\sigma_X], \quad (13)$$

where $\Delta = \omega_0 - \omega$, $V(X) = \sqrt{\Delta^2 + 4\Omega^2 \cos^2(kX)}$, $\cos\theta(X) = \Delta/V(X)$ and $\sin\theta(X) = 2\Omega \cos(kX)/V(X)$.

In the limit of large detuning, i.e. $\Delta \gg 2\Omega \cos(kX)$, the effective Hamiltonian remains

$$H = \hbar\Delta\sigma_Z + \left[\frac{2\hbar\Omega^2 \cos^2(kX)}{\Delta} \right] \sigma_Z. \quad (14)$$

The atomic state vector can be written as

$$\langle X|\Psi(t)\rangle = a(X,t)|e\rangle + b(X,t)|g\rangle, \quad (15)$$

where $|g\rangle$, and $|e\rangle$ are the ground and excited internal atomic states, while a , and b are probability amplitudes.

We assume the atoms initially in their ground state with a Gaussian wavefunction

$$a(x,0) = 0, \quad (16)$$

$$b(x,0) = (\pi\sigma^2)^{-1/4} \exp\left[-\frac{x^2}{2\epsilon_x}\right], \quad (17)$$

where $x = kX$ and ϵ_x is the transverse position spread of the input beam.

The Schrödinger equations are

$$\frac{d}{dt}a = i \left[\frac{\Delta}{2} + \frac{\Omega^2 \cos^2 x}{\Delta} \right] a, \quad (18)$$

$$\frac{d}{dt}b = -i \left[\frac{\Delta}{2} + \frac{\Omega^2 \cos^2 x}{\Delta} \right] b, \quad (19)$$

Of course, it results $a(x,t) = 0$, while the solution for b can be written in terms of the Bessel functions of the first kind J_n [10], as

$$b(x, t) = \exp \left[i \left(\frac{\Delta}{2} + \frac{\Omega^2}{2\Delta} \right) t \right] \sum_{n=-\infty}^{+\infty} i^n J_n \left(\frac{\Omega^2 t}{2\Delta} \right) e^{2inx} b(x, 0). \quad (20)$$

Taking the Fourier transform of this relationship shows the effect in the momentum space

$$\tilde{b}(p_x, t) = \exp \left[i \left(\frac{\Delta}{2} + \frac{\Omega^2}{2\Delta} \right) t \right] \sum_{n=-\infty}^{+\infty} i^n J_n \left(\frac{\Omega^2 t}{2\Delta} \right) \tilde{b}(p_x - 2n, 0), \quad (21)$$

where \tilde{b} denotes the Fourier transform of b , and $p_x = P_X/\hbar k$. The scattered ground state wavefunction is a superposition of Gaussian modulated plane waves with momentum $2n\hbar k$. The momentum transferred from the field to the atom is an even multiplies of $\hbar k$ corresponding to the absorption of a photon from the $(+k)$ component followed by induced emission in the $(-k)$ component of the standing wave.

The final output momentum probability, which can be measured, is composed of a comb of images of the initial momentum distribution

$$w(p_x, T) = \left| \tilde{b}(p_x, T) \right|^2 = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} i^{(n-m)} J_n(T/4) J_m(T/4) \tilde{b}(p_x - 2n, 0) \tilde{b}(p_x - 2m, 0), \quad (22)$$

where $T = 2\Omega^2 t/\Delta$. In order to resolve the peaks it is necessary to have a narrow initial momentum spread, i.e. $\epsilon_x > 1$.

IV. EFFECTS OF TIME FLUCTUATIONS

The atomic wave packet has a finite position spread also in the longitudinal direction (Z in Fig.1). For the sake of simplicity we assume $\epsilon_z = \epsilon_x = \epsilon = k\epsilon_Z = k\epsilon_X$. The non-zero value of ϵ_z give rise to an uncertainty in the arrival time [6] of the atoms at the end of the cavity, i.e. at the time T of Eq.(22). We can account for this uncertainty by considering the time as a random variable and use the quantum mechanical consistent approach described in Section II [11]. Then, accordingly to Eq.(1), we have to do the following replacement

$$w(p_x, T) \equiv \text{Tr} \{ p_x \rho(T) \} \longrightarrow \overline{w}(p_x, T) \equiv \text{Tr} \{ p_x \overline{\rho}(T) \}. \quad (23)$$

From Eqs.(21) and (5), it results

$$\overline{w}(p_x, T) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} i^{(n-m)} I_{n,m}(T) \tilde{b}(p_x - 2n, 0) \tilde{b}(p_x - 2m, 0), \quad (24)$$

where

$$I_{n,m}(T) = \int_0^\infty dT' J_n(T'/4) J_m(T'/4) \frac{1}{T} \frac{e^{-T'/T}}{\Gamma(T/T)} (T'/T)^{T/T-1}, \quad (25)$$

with $T = 2\Omega^2 \tau/\Delta$, and the integral given explicitly in the appendix.

The output momentum distribution is shown in Fig.2 for several values of T . Notice, that the chosen interaction time T implies the deflection of all atoms in the ideal situation a), where no peak is present at $p_x = 0$. However, the effect of intrinsic decoherence is to reduce the exchange of momentum between atoms and field, then to leave the atoms undeflected. In fact the central peak increases from b) to c) while the others decrease.

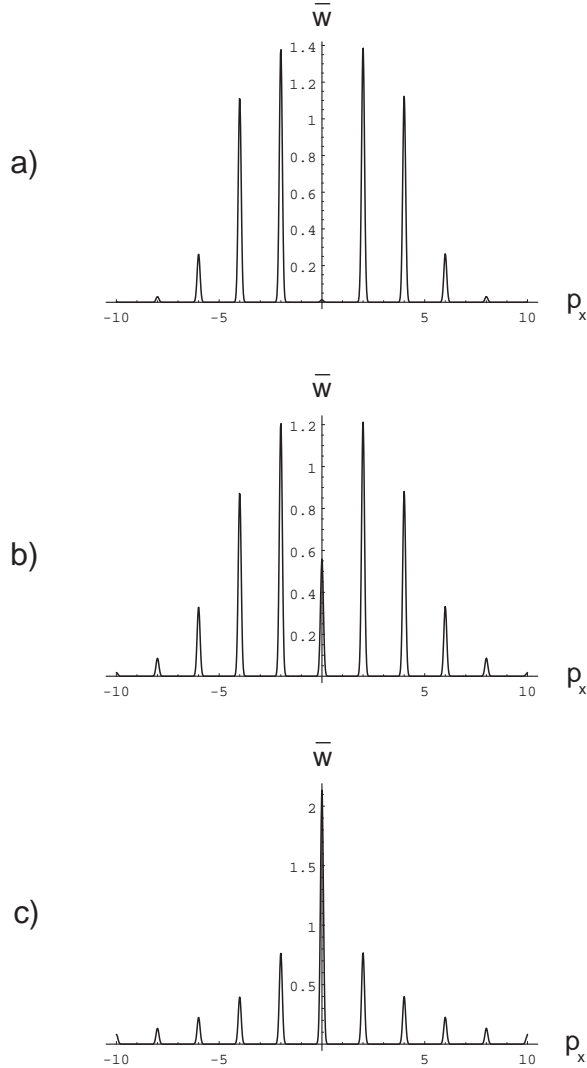


FIG. 2. Momentum distribution \bar{w} for $T = 0$ a); $T = 1$ b); and $T = 10$ c). The values of other parameters are: $T = 10$, and $\epsilon = 10$.

V. CONCLUSIONS

We know that \mathcal{T} is related to the time parameter τ . The latter represents the time uncertainty in the interaction process. On the other hand, the uncertainty in localizing temporally a wave packet at a given space point, which give rise to the time energy uncertainty relation [6], may be considered as $\delta Z/v$, where δZ is the width of the packet and v its group velocity [6]. Then, we may assume [2]

$$\tau = \frac{1}{\langle P_Z \rangle / M} \sqrt{\epsilon_Z^2 + \left(\frac{\hbar}{2M} \right)^2 \frac{t^2}{\epsilon_Z^2} + \left(\frac{1}{M} \right)^2 \mathcal{E}_{P_Z}^2 t^2}, \quad (26)$$

where $\langle P_Z \rangle / M$ is the mean velocity of the atoms along the Z direction, while the square root term represents the width of the packet along the Z direction. The first two terms inside the square root of Eq.(26) describe the well known free particle wave packet spread (during the interaction time t), while the last term is due to the classical momentum spread. In this case τ (hence \mathcal{T}) becomes time dependent, but the positive map described by Eqs.(1), (5), can be maintained by dropping the semigroup property.

Of course, if in the above expression (26) \mathcal{E}_{P_Z} is the dominant term, we only have classical decoherence effect, essentially due to the thermal spread of momentum. However, even if such effect is eliminated, decoherence will be

still present having a purely quantum nature. Moreover, it will be intrinsic to the system, and not due to external degrees of freedom.

Effects similar to those of Fig.(2) can be recognized in the experimental data of Ref. [5], but mainly due to (classical) velocity spread. Nevertheless, more recently, cooling techniques offer the possibility to have almost monochromatic atomic beam (e.g. $\delta\langle P_Z\rangle/\langle P_Z\rangle = 10^{-3}$ with $\langle P_Z\rangle/M = 10^3 \text{ ms}^{-1}$ [12]). Then, it would be possible to achieve minimum uncertainty states for the atomic wave packet, with e.g. $\epsilon_Z \approx 10^{-11} \text{ m}$ [12]. This means that the dominant term in Eq.(26), once fixed the interaction time $t \approx 10^{-9} \text{ s}$ [5], becomes the Schrodinger spread, i.e. the second term in the square root. That gives $\mathcal{T} \approx 1$ which is promising to test the intrinsic quantum nature of decoherence as can be seen from Fig.(2). On the other hand, the opposite limit, where ϵ_Z is dominant in Eq.(26) could be investigated as well with slow atoms at large de Broglie wavelength [13].

In conclusion, we have studied intrinsic decoherence arising from the interaction time fluctuations in atomic diffraction by standing light wave. The main limitation to observe such effect relies on the necessity to suppress classical fluctuations, however, the technological achievements in atom optics, atom lasers [14] at last, make the possibility to observe *intrinsic quantum decoherence* realistic. Finally, it is worth interesting to also apply the above theory to the problem of atomic wave diffraction in time [15]; this is planned for future work.

APPENDIX

The integral (25) can be written explicitly [16] as follows

$$I_{n,m}(T) = I_{|n|,|m|}(T) (-)^{(|n|-n+|m|-m)/2}, \quad (27)$$

where

$$I_{|n|,|m|}(T) = \frac{\mathcal{T}^{(|n|+|m|)} \Gamma(|n|+|m|+\frac{T}{\mathcal{T}}) {}_4F_3\left(\mathbf{u}, \mathbf{v}, -\frac{\mathcal{T}^2}{4}\right)}{2^{3(|n|+|m|)} \Gamma(1+|n|) \Gamma(1+|m|) \Gamma(\frac{T}{\mathcal{T}})}, \quad (28)$$

with F the generalized hypergeometric function [10] whose vectors \mathbf{u} and \mathbf{v} are

$$\mathbf{u} \equiv \left(\frac{1}{2} + \frac{|n|}{2} + \frac{|m|}{2}, \frac{1}{2} + \frac{|n|}{2} + \frac{|m|}{2}, \frac{|n|}{2} + \frac{|m|}{2} + \frac{T}{2\mathcal{T}}, \frac{1}{2} + \frac{|n|}{2} + \frac{|m|}{2} + \frac{T}{2\mathcal{T}} \right), \quad (29)$$

$$\mathbf{v} \equiv (1+|n|, 1+|m|, 1+|n|+|m|). \quad (30)$$

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